

Constraints on string resonance amplitudes

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We perform a global analysis of the tree-level open-string amplitudes in the limit $s \ll M_S^2$. Based on the present data from the Tevatron, HERA, and LEP 2, we set a lower limit on the string scale $M_S \geq 0.69\text{--}1.96$ TeV at 95% confidence level for the Chan-Paton factors $|T| = 0\text{--}4$. We also estimate the expected sensitivities at the CERN LHC, which can be as high as 19 TeV for $|T| = 4$.

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I. INTRODUCTION

String theory [1] provides a consistent framework for the unification of gravity with the standard model (SM) gauge interactions. The string scale M_S is naturally close to the quantum gravity scale $M_{pl} \approx 10^{19}$ GeV. However, Arkani-Hamed, Dimopoulos, and Dvali (ADD) [2] proposed an alternative to this scenario. They assumed the space-time is more than 4 dimensions with the SM particles living on a D3-brane. While the electromagnetic, strong, and weak forces are confined to this brane, gravity can propagate in the extra dimensions. The effective fundamental Planck scale is of the order of TeV with large extra dimensions as large as mm . TeV scale string theories are also possible with the advance of D-branes [3,4], thus, making it possible for experiments to test and probe the theory at high-energy colliders [5].

In the simplest open-string model [6–8], it is assumed that all SM particles are identified as open strings confined to a D3-brane universe while a graviton is a closed string propagating freely in the bulk. One novel feature of the open-string model is the appearance of string resonances in the scattering of particles. Earlier works on phenomenological studies [6–10] based on open-string amplitudes have been performed, which reduce to SM tree-level amplitudes at low energies. When $s > M_S^2$, clear resonances can be observed in experiments, in particular, the mass spectrum of the resonances is given by $\sqrt{n}M_S$ ($n = 1, 2, 3, \dots$), which is very different from the Kaluza-Klein (KK) states in the ADD [2] or Randall-Sundrum [11] models. On the other hand, when $s < M_S^2$ there would not appear resonances but may experience virtual interference effects from the string resonances. Effectively, the virtual effect is similar to a contact interaction scaling as $1/M_S^4$. Note that the usual 4-fermion contact interaction due to the exchange of new gauge bosons or leptoquarks has the leading effect of $1/\Lambda^2$, where Λ is the scale of new physics [12]. Bounds on 4-fermion contact interactions induced by string resonances have been estimated based on the analysis on $1/\Lambda^2$ contact interaction [9]. Although the approxi-

mation is valid, we shall show in this work that the effect is more complicated than just the simple substitution $1/\Lambda^2 \rightarrow 1/M_S^4$.

The major improvement from previous work is as follows. We start directly from open-string amplitudes and perform a global analysis instead of using the result of an analysis on contact interaction [13]. The global data include Drell-Yan production at the Tevatron, HERA charged and neutral-current deep-inelastic scattering, total hadronic, and leptonic $e^+e^- \rightarrow e^+e^-$, $\mu^+\mu^-$, $\tau^+\tau^-$ cross section at LEP 2. Some of the data (CDF, HERA, and LEP 2) have been updated since the summer of 2004. We shall see that Drell-Yan production, due to large invariant mass data, provides the strongest constraint among the global data. By combining all data, the string scale M_S must be larger than 0.69–1.96 TeV for $|T| = 0\text{--}4$ at 95% confidence level (C.L.).

The organization of the paper is as follows. In the next section, we describe the tree-level open-string amplitude and its low-energy approximation. In Sec. III we describe the high-energy data sets that we used in this analysis. In Sec. IV, we present our results on the fits and limits. In Sec. V, we estimate the sensitivity at the LHC. We summarize in Sec. VI.

II. OPEN-STRING AMPLITUDES

Our construction of tree-level open-string amplitudes follows the same approach as that outlined in Refs. [7–10]. We briefly summarize here. Tree-level open-string amplitudes represent the scattering of massless SM particles as the zeroth string modes. The SM is embedded in a type IIB string theory whose 10-dimensional space has six of the dimensions compactified on a torus with a common period $2\pi R$. There are \mathcal{N} coincident D3-branes on which open strings may end. This model is constructed in such a way that the string amplitudes reduce to the SM amplitudes in the low-energy limit, as well as demonstrates the effects of string resonances.

The 4-point tree-level open-string amplitude is given as a sum of ordered amplitudes multiplied by group theory Chan-Paton factors and can be expressed generically as [6–8]

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$$\begin{aligned} \mathcal{A}_{\text{string}} = & S(s, t)A_{1234}T_{1234} + S(t, u)A_{1324}T_{1324} \\ & + S(u, s)A_{1243}T_{1243}, \end{aligned} \quad (1)$$

where A_{ijkl} 's are kinematic parts of the amplitude. The Mandelstam variables are denoted by s , t , and u . T_{ijkl} 's are the Chan-Paton factors [14] that involve traces over the group representation matrices λ . For example,

$$T_{1234} = \text{Tr}(\lambda_1 \lambda_2 \lambda_3 \lambda_4) + \text{Tr}(\lambda_4 \lambda_3 \lambda_2 \lambda_1).$$

Typically, with normalization $\text{Tr}(\lambda^a \lambda^b) = \delta^{ab}$ the Chan-Paton factors are in the range of -4 to 4 . Since a complete string model construction for the electroweak interaction of the SM is unavailable, Chan-Paton factors are taken as free parameters in our calculation. $S(s, t)$ is essentially the Veneziano amplitude [15]

$$S(s, t) = \frac{\Gamma(1 - \alpha' s) \Gamma(1 - \alpha' t)}{\Gamma(1 - \alpha' s - \alpha' t)}, \quad (2)$$

where the Regge slope $\alpha' = M_S^{-2}$, and $S \rightarrow 1$ as either s/M_S^2 or $t/M_S^2 \rightarrow 0$.

We start with Drell-Yan production of a pair of leptons. In the massless limit of the fermions we label their chirality $\alpha, \beta = L, R$. The tree-level open-string amplitude is

$$\begin{aligned} \mathcal{A}_{\text{string}}(q_L \bar{q}_L \rightarrow \ell_R \bar{\ell}_R) = & ig_s^2 \left[T_{1234} S(s, t) \frac{t}{s} \right. \\ & + T_{1324} S(t, u) \frac{t}{u} + T_{1243} S(u, s) \\ & \left. \times \left(-\frac{t}{s} - \frac{t}{u} \right) \right]. \end{aligned} \quad (3)$$

The corresponding standard model amplitude is

$$\mathcal{A}_{\text{SM}}(q_L \bar{q}_L \rightarrow \ell_R \bar{\ell}_R) = ig^2 \frac{t}{s} \mathcal{F}_{RL}, \quad (4)$$

where

$$\mathcal{F}_{\alpha\beta} = 2Q_\ell Q_q \sin^2 \theta_w + \frac{s}{s - m_Z^2} \frac{2g_\alpha^\ell g_\beta^q}{\cos^2 \theta_w}. \quad (5)$$

Here $g_L^f = T_{3f} - Q_f \sin^2 \theta_w$, $g_R^f = -Q_f \sin^2 \theta_w$, Q_f is the electric charge of the fermion f in units of proton charge, and the $\text{SU}(2)_L$ coupling $g = e/\sin \theta_w$. Identifying the string coupling with the gauge coupling $g_S = g$ and matching the Chan-Paton factor T_{ijkl} as

$$T_{1243} = T_{1324} \equiv T, \quad T_{1234} = T + 1 \quad (6)$$

one can demand the string amplitude expression in Eq. (3) to reproduce the standard model amplitude in the low-energy limit. Since the string amplitude describes massless particle scattering, the masses of the W and Z gauge bosons are introduced by hand. So Eq. (3) becomes

$$\begin{aligned} \mathcal{A}_{\text{string}}(q_L \bar{q}_L \rightarrow \ell_R \bar{\ell}_R) = & ig^2 S(s, t) \frac{t}{s} \mathcal{F}_{RL} \\ & + ig^2 T \frac{t}{us} f(s, t, u), \end{aligned} \quad (7)$$

$$f(s, t, u) \equiv uS(s, t) + sS(t, u) + tS(u, s). \quad (8)$$

The amplitude for $q_R \bar{q}_R \rightarrow \ell_L \bar{\ell}_L$ process is obtained from Eq. (7) by changing \mathcal{F}_{RL} to \mathcal{F}_{LR} . As for the $q_R \bar{q}_R \rightarrow \ell_R \bar{\ell}_R$ and $q_L \bar{q}_L \rightarrow \ell_L \bar{\ell}_L$ processes, we use the corresponding \mathcal{F}_{RR} and \mathcal{F}_{LL} , respectively, and interchange $t \leftrightarrow u$ in Eq. (7).

The factor $\Gamma(1 - s/M_S^2)$ in the Veneziano amplitude $S(s, t)$ develops poles at $s = nM_S^2$ ($n = 1, 2, 3, \dots$), implying resonance states with masses $\sqrt{n}M_S$. In the limit $M_S \gg \sqrt{s}, \sqrt{|t|}, \sqrt{|u|}$

$$S(s, t) \approx 1 - \frac{\pi^2}{6} \frac{st}{M_S^4}, \quad S(t, u) \approx 1 - \frac{\pi^2}{6} \frac{tu}{M_S^4}, \quad (9)$$

$$S(u, s) \approx 1 - \frac{\pi^2}{6} \frac{us}{M_S^4},$$

$$f(s, t, u) \approx -\frac{\pi^2}{2} \frac{stu}{M_S^4}. \quad (10)$$

Therefore the amplitude squared for the Drell-Yan process is given by

$$\begin{aligned} \sum |\mathcal{M}|^2 = & 4u^2(|M_{LL}^{\ell q}(s)|^2 + |M_{RR}^{\ell q}(s)|^2) + 4t^2(|M_{LR}^{\ell q}(s)|^2 \\ & + |M_{RL}^{\ell q}(s)|^2), \end{aligned}$$

where

$$\begin{aligned} M_{\alpha\alpha}^{\ell q}(s) = & e^2 \left[\left(\frac{Q_\ell Q_q}{s} + \frac{g_\alpha^\ell g_\alpha^q}{\sin^2 \theta_w \cos^2 \theta_w} \frac{1}{s - M_Z^2} \right) \right. \\ & \left. \times \left(1 - \frac{\pi^2 su}{6M_S^4} \right) - \frac{\pi^2 uT}{4 \sin^2 \theta_w M_S^4} \right], \quad \alpha = L, R, \end{aligned} \quad (11)$$

$$\begin{aligned} M_{\alpha\beta}^{\ell q}(s) = & e^2 \left[\left(\frac{Q_\ell Q_q}{s} + \frac{g_\alpha^\ell g_\beta^q}{\sin^2 \theta_w \cos^2 \theta_w} \frac{1}{s - M_Z^2} \right) \right. \\ & \left. \times \left(1 - \frac{\pi^2 st}{6M_S^4} \right) - \frac{\pi^2 tT}{4 \sin^2 \theta_w M_S^4} \right] \begin{matrix} \alpha, \beta = L, R \\ \alpha \neq \beta \end{matrix}. \end{aligned} \quad (12)$$

Based on the above formulas the square of amplitudes for other processes can be obtained by crossing the Mandelstam variables and replacing the corresponding gauge boson.

III. HIGH-ENERGY PROCESSES AND DATA SETS

Before describing the data sets used in our analysis, let us first specify certain important aspects of the analysis technique. Since the next-to-leading order calculations do not exist for the new interaction, we use leading order (LO) calculations for contributions from both the SM and the new interactions, for consistency. However, in many cases, e.g. in the analysis of precision electroweak parameters, it is important to use the best available calculations of their SM values, as in many cases data are sensitive to the next-to-leading and sometimes even to higher-order corrections. Therefore, we normalize our leading order calculations to either the best calculations available, or to the low- Q^2 region of the data set, where the contribution from the string resonances is expected to be vanishing. This is equivalent to introducing a Q^2 -dependent K factor and using the same K factor for both the SM contribution and the effects of string resonances. The details of this procedure for each data set are given in the corresponding section. Wherever parton distribution functions (PDFs) are needed, we use the CTEQ5L (leading order fit) set [16]. The reason to use the LO PDF set is that LO PDFs are extracted using LO cross section calculations, thus making them consistent with our approach.

A. Drell-Yan production at the Tevatron

Both CDF [17] and D0 [18] measured the differential cross section $d\sigma/dM_{\ell\ell}$ for Drell-Yan production, where $M_{\ell\ell}$ is the invariant mass of the lepton pair. (CDF analyzed data in both the electron and muon channels; D0 analyzed only the electron channel.) The differential cross section, including the contributions from the string resonances, is given by

$$\frac{d^2\sigma}{dM_{\ell\ell}dy} = K \frac{M_{\ell\ell}^3}{72\pi s} \sum_q f_q(x_1) f_{\bar{q}}(x_2) (|M_{LL}^{\ell q}(\hat{s})|^2 + |M_{LR}^{\ell q}(\hat{s})|^2 + |M_{RL}^{\ell q}(\hat{s})|^2 + |M_{RR}^{\ell q}(\hat{s})|^2),$$

where $M_{\alpha\beta}^{\ell q}$ is given by Eqs. (11) and (12), $\hat{s} = M_{\ell\ell}^2$, \sqrt{s} is

the center-of-mass energy in the $p\bar{p}$ collisions, $M_{\ell\ell}$ and y are the invariant mass and the rapidity of the lepton pair, respectively, and $x_{1,2} = (M_{\ell\ell}/\sqrt{s})e^{\pm y}$. The variable y is integrated numerically to obtain the invariant mass spectrum. The QCD K factor is given by $K = 1 + [\alpha_s(\hat{s})/2\pi] \frac{4}{3} [1 + (4\pi^2/3)]$. We scale this tree-level SM cross section by normalizing it to the Z-peak cross section measured with the data by a scale factor C (C is very close to 1 and numerically C is equal to 1.14, 1.13, 1.00 for the CDF e^- , CDF μ^- , and D0 e^- samples, respectively.) The cross section σ used in the fitting procedure is given by

$$\sigma = C(\sigma_{\text{SM}} + \sigma_{\text{interf}} + \sigma_{\text{string}}), \quad (13)$$

where σ_{interf} is the interference term between the SM and the string resonance amplitudes and σ_{string} is the cross section due to the string resonance interactions only. When normalizing to the low-energy data, we neglect the possible contribution from the string resonances, as it is much smaller than the experimental uncertainty on the data that we use.

B. HERA neutral- and charged-current data

ZEUS [19,20] and H1 [21] have published results on the neutral-current (NC) and charged-current (CC) deep-inelastic scattering (DIS) in e^+p collisions at $\sqrt{s} \approx 319$ and 318 GeV, respectively. The data sets collected by H1 and ZEUS correspond to integrated luminosities of 65.2 (H1), 60.9 (ZEUS CC), and 63.2 (ZEUS NC) pb^{-1} . ZEUS [22,23] and H1 [24] have also published NC and CC analysis for the data collected in e^-p collisions at $\sqrt{s} \approx 318$ and 320 GeV, respectively, with integrated luminosities of 16.4 (ZEUS CC), 15.9 (ZEUS NC), and 16.4 (H1) pb^{-1} .

We used the single differential cross section $d\sigma/dQ^2$ presented by ZEUS [19,20] and double differential cross section $d^2\sigma/dxdQ^2$ published by H1 [21]. The double differential cross section for NC DIS in the e^+p collisions, including the effect of the string resonances, is given by

$$\begin{aligned} \frac{d^2\sigma}{dxdQ^2}(e^+p \rightarrow e^+X) = \frac{1}{16\pi} \Big\{ \sum_q f_q(x) [(1-y)^2 (|M_{LL}^{eq}(t)|^2 + |M_{RR}^{eq}(t)|^2) + |M_{LR}^{eq}(t)|^2 + |M_{RL}^{eq}(t)|^2] + \sum_{\bar{q}} f_{\bar{q}}(x) [|M_{LL}^{eq}(t)|^2 + |M_{RR}^{eq}(t)|^2 + (1-y)^2 (|M_{LR}^{eq}(t)|^2 + |M_{RL}^{eq}(t)|^2)] \Big\}, \end{aligned} \quad (14)$$

where $Q^2 = sxy$ is the square of the momentum transfer and $f_{q/\bar{q}}(x)$ are parton distribution functions. The reduced amplitudes $M_{\alpha\beta}^{eq}$ are given by Eqs. (11) and (12). The double differential cross section for CC DIS, including the effect of string resonances, can be written as

$$\begin{aligned} \frac{d^2\sigma}{dx dQ^2}(e^+p \rightarrow \bar{\nu}X) = & \frac{g^4}{64\pi} \left\{ \left| \frac{1}{-Q^2 - M_W^2} \left(1 - \frac{\pi^2 tu}{6M_S^4} \right) - \frac{\pi^2 uT}{2M_S^4} \right|^2 [(1-y)^2(d(x) + s(x))] \right. \\ & \left. + \left| \frac{1}{-Q^2 - M_W^2} \left(1 - \frac{\pi^2 st}{6M_S^4} \right) - \frac{\pi^2 sT}{2M_S^4} \right|^2 [\bar{u}(x) + \bar{c}(x)] \right\}, \end{aligned} \quad (15)$$

where $d(x)$, $s(x)$, $\bar{u}(x)$, and $\bar{c}(x)$ are the parton distribution functions. The single differential cross section $d\sigma/dQ^2$ is obtained from the above equations by integrating over x . The cross section in the e^-p collisions can be obtained by interchanging ($LL \leftrightarrow LR$, $RR \leftrightarrow RL$) in Eq. (14) and by interchanging ($q(x) \leftrightarrow \bar{q}(x)$) in Eq. (15).

We normalize the tree-level SM cross section to that measured in the low- Q^2 ($Q^2 \leq 2000 \text{ GeV}^2$) region. The cross section used in the fitting procedure is then obtained similar to that in Eq. (13). Numerically, the normalization factor $C = 1.00, 1.00, 0.94, 1.07, 1.00, 1.00, 1.00, 1.00$ for ZEUS e^-p NC, CC, e^+p NC, CC, and H1 e^-p NC, CC, e^+p NC, CC samples, respectively.

C. LEP 2 Data

We analyze the LEP 2 observables sensitive to the effects of the string resonances, including hadronic and leptonic cross sections. The LEP Electroweak Working Group combined the $q\bar{q}$, $\mu^+\mu^-$, and $\tau^+\tau^-$ data from all four LEP collaborations [25] for the machine energies between 130 and 207 GeV. We use the following quantities in our analysis: (i) total hadronic cross sections, (ii) total $\mu^+\mu^-$, $\tau^+\tau^-$ cross sections. We take into account the correlations of the data points in each data set as given by [25]. For the Bhabha scattering cross section $\sigma(e^+e^- \rightarrow e^+e^-)$, we use various data sets from individual experiments [26,27].

The angular distribution for $e^-e^+ \rightarrow f\bar{f}$ ($f = q, e, \mu, \tau$) is given by

$$\begin{aligned} \frac{d\sigma}{d\cos\theta} = & \frac{N_f s}{128\pi} \{ (1 + \cos\theta)^2 (|M_{LL}^{ef}(s)|^2 + |M_{RR}^{ef}(s)|^2) \\ & + (1 - \cos\theta)^2 (|M_{LR}^{ef}(s)|^2 + |M_{RL}^{ef}(s)|^2) \\ & + \delta_{ef} [(1 + \cos\theta)^2 (|M_{LL}^{ef}(s) + M_{LL}^{ef}(t)|^2 \\ & + |M_{RR}^{ef}(s) + M_{RR}^{ef}(t)|^2 - |M_{LL}^{ef}(s)|^2 \\ & - |M_{RR}^{ef}(s)|^2) + 4(|M_{LR}^{ef}(t)|^2 + |M_{RL}^{ef}(t)|^2)] \}, \end{aligned}$$

where $N_f = 1$ (3) for ℓ (q), and $M_{\alpha\beta}^{ef}$ is given by Eqs. (11) and (12). The additional terms for $f = e$ arise from the t , u -channel exchange diagrams.

To minimize the uncertainties from higher-order corrections, we normalize the tree-level SM calculations to the next-to-leading order cross section, quoted in the corresponding experimental papers. We then scale our tree-level results, including contributions from the Z , γ , and string resonances with this normalization factor, similar to Eq. (13). When fitting the angular distribution, we fit to

the shape only, and treat the normalization as a free parameter of the fit.

IV. CONSTRAINTS FROM HIGH-ENERGY EXPERIMENTS

In the previous section, we have described the data sets from various high-energy experiments used in our analysis. Based on the above individual and combined data sets, we perform a fit to the sum of the SM prediction and the contribution of the string resonances, normalizing our tree-level cross section to the best available higher-order calculations, as explained above. As seen from Eqs. (11) and (12), the effects of the string resonances always enter the equations in the form $1/M_S^4$. Therefore, we parametrize these effects with a single fit parameter η :

$$\eta \equiv \frac{1}{M_S^4}.$$

In most cases, the differential cross sections in the presence of the string resonances are bilinear in η . We use MINUIT to minimize the χ^2 .

The best-fit values of η for each individual data set and their combinations are shown in Table I. In all cases, the preferred values from the fit are consistent with zero, and therefore we proceed with setting limits on η . The one-sided 95% C.L. upper limit on η is defined as

$$0.95 = \frac{\int_0^{\eta_{95}} d\eta P(\eta)}{\int_0^\infty d\eta P(\eta)}, \quad (16)$$

where $P(\eta)$ is the fit likelihood function given by

$$P(\eta) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-(\chi^2(\eta) - \chi_{\min}^2)/2). \quad (17)$$

The corresponding upper 95% C.L. limits on η and lower 95% C.L. limits on M_S are also shown in Table I. The combined limit is as high as 1.96 (1.92) TeV for $T = 4$ (-4). Note that even for $T = 0$, there is still some stringy effects ($1/M_S^4$), as shown in Eqs. (11) and (12). It is clear from the results of Table I that the dominant set of data affecting the limit on M_S is the Drell-Yan data. This is simply because the effect of string resonances scales like $1/M_S^4$. We show the effect of string resonances on the invariant mass spectrum in Fig. 1.

The limits obtained in this work have shown considerable improvement over those in Ref. [9], which was based on a contact interaction analysis [13], and similar to those in Ref. [10], which was based on the search for Z' at the Tevatron. The improvements of the limits of this work over

TABLE I. Best-fit values of $\eta = 1/M_S^4$ and the 95% C.L. upper limits on η for individual data set and combinations. Corresponding 95% C.L. lower limits on M_S are also shown. By default we use $T = 4$ in (a). We show in (b) limits for all combined data with different T .

(a)			
	η (TeV ⁻⁴)	η_{95} (TeV ⁻⁴)	M_S^{95} (TeV)
TEVATRON:			
Drell-Yan	$-0.013^{+0.070}_{-0.044}$	0.074	1.92
HERA:			
NC	$0.311^{+0.208}_{-0.192}$	0.71	1.09
CC	$1.318^{+1.186}_{-1.223}$	3.33	0.74
HERA combined	$0.336^{+0.210}_{-0.192}$	0.73	1.08
LEP 2:			
Hadronic cross section and angular distribution	$-1.17^{+0.34}_{-0.28}$	2.92	0.76
$ee, \mu\mu, \tau\tau$ cross section and angular distribution	$-0.122^{+0.097}_{-0.098}$	0.12	1.68
LEP 2 combined	$-0.156^{+0.098}_{-0.098}$	0.11	1.72
All combined ($T = 4$)	$-0.023^{+0.059}_{-0.034}$	0.066	1.96
(b)			
All combined ($T = 1$)	$-0.044^{+0.227}_{-0.166}$	0.27	1.38
All combined ($T = 0$)	$2.28^{+1.27}_{-1.27}$	4.38	0.69
All combined ($T = -1$)	$0.138^{+0.114}_{-0.193}$	0.29	1.35
All combined ($T = -4$)	$0.028^{+0.031}_{-0.055}$	0.072	1.92

those using the contact interaction analysis were partially because the deviation term in the string amplitudes is energy dependent. The highest invariant mass bin in the CDF and D0 data sets is up to 999 or 1000 GeV [17,18]. This value is higher than the typical value of \hat{s} used in converting the limits from contact interaction [9].

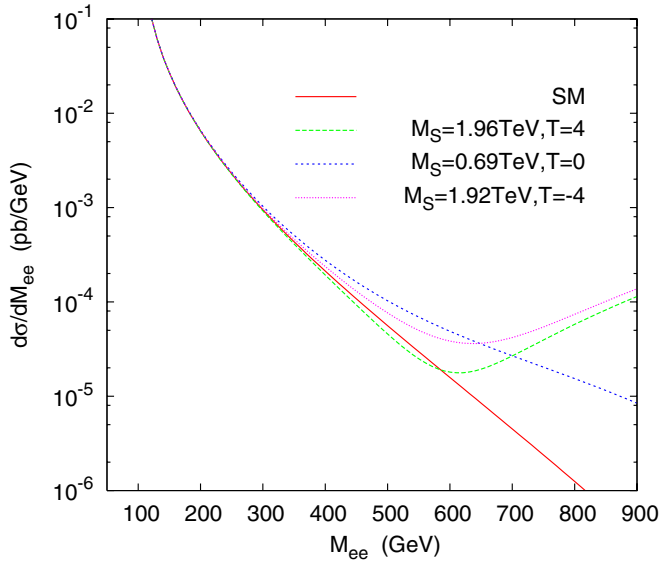


FIG. 1 (color online). The differential distribution $d\sigma/dM_{ee}$ for Drell-Yan production at the Tevatron (run I) for the low scale open-string model and for the SM.

Therefore, the limits in this work improve by some extent. Also, it is partially because we used more updated data from the Tevatron, HERA, and LEP 2.

V. SENSITIVITY AT THE LHC

We expect that at the LHC the best channel to probe the string resonances is Drell-Yan production. We assume that the approximation $M_S^2 \gg \hat{s}, |\hat{t}|, |\hat{u}|$ is still valid at the LHC. Therefore, the reduced amplitudes of Eqs. (11) and (12) can be used and tested by a direct comparison at the LHC.

It was shown in Ref. [28] that using the double differential distribution $d^2\sigma/M_{\ell\ell}d\cos\theta$ can increase the sensitivity to the KK states of the graviton compared to the use of single differential distributions. Similarly, we expect this to be the case for the string resonances. The double differential cross section for Drell-Yan production, including the interactions of the γ , Z , and string resonances, is given by

$$\begin{aligned} \frac{d^3\sigma}{dM_{\ell\ell}dyd\cos\theta^*} &= K \sum_q \frac{M_{\ell\ell}^3}{192\pi s} f_q(x_1) f_{\bar{q}}(x_2) [(1 + \cos\theta^*)^2 \\ &\quad \times (|M_{LL}^{eq}(\hat{s})|^2 + |M_{RR}^{eq}(\hat{s})|^2) + (1 \\ &\quad - \cos\theta^*)^2 (|M_{LR}^{eq}(\hat{s})|^2 + |M_{RL}^{eq}(\hat{s})|^2)], \end{aligned}$$

where $M_{\alpha\beta}^{eq}$ are given by Eqs. (11) and (12), θ^* is the scattering angle in the center-of-mass frame of the initial partons, $\hat{s} = M_{\ell\ell}^2$, $dx_1 dx_2 = (2M_{\ell\ell}/s) dM_{\ell\ell} dy$, and $x_{1,2} = M_{\ell\ell} e^{\pm y}/\sqrt{s}$.

TABLE II. Sensitivity to the parameter $\eta = 1/M_S^4$ at the LHC, using the dilepton channels. The corresponding 95% C.L. lower limits on M_S are also shown.

LHC (14 TeV, 100 fb ⁻¹) dilepton	η_{95} (TeV ⁻⁴)	95% C.L. lower limit on M_S (TeV)
$T = 4$	8.49×10^{-6}	18.5
$T = 1$	3.71×10^{-5}	12.8
$T = 0$	5.79×10^{-4}	6.4
$T = -1$	3.42×10^{-5}	13.0
$T = -4$	7.95×10^{-6}	18.8

We briefly describe the procedures here. (i) We calculate the SM cross section in each bin. (ii) In each bin, we obtain the expected number of events by multiplying the SM cross section by the integrated luminosity. (iii) In each bin, we generate the number of events according to the expected number of events. If the expected number of events is less than 100, we used the Poisson statistics; otherwise we used the Gaussian. (iv) After generating the data set, we fit the data set to η using Eq. (13). Then we evaluate the 95% C.L. lower limit on M_S implied by the data. (v) We repeat the above procedure for 1000 times. Then we histogram the limit obtaining each time. We take the sensitivity to be the medium (501st) of the histogram.

In each simulation, we always normalize to the Z-peak data, by which a lot of systematic uncertainties are eliminated. We have used an efficiency of 90% and a rapidity coverage of $|\eta| < 2.0$ for lepton detection. We also used a single experiment with combined e, μ samples. The sensitivity may be lowered by about 10% if some level of systematic uncertainties is used in calculating the χ^2 [28]. The sensitivity, at the 95% C.L., to M_S at the LHC (100 fb⁻¹) with different Chan-Paton factors is given in Table II. The sensitivity reach is as high as 19 TeV for $|T| = 4$. Note that the sensitivity obtained in Ref. [10] is somewhat lower than the sensitivity obtained in this paper, because their analysis is based on the search for a single resonance state while ours is based on the deviation in the

whole invariant mass spectrum. Therefore, statistically we can obtain a higher sensitivity.

VI. CONCLUSIONS

The ultimate consequence of a low string scale is the testable string scattering amplitudes at the hadron collider. The scattering of particles shows the stringy nature. The most obvious sign is the presence of string resonances. Even if the energy is not high enough, some deviations from the SM due to the string resonances are still expected. For example, the tree-level open-string amplitude for dilepton production at low energy can be expressed generically as

$$\mathcal{A}_{\text{string}} \sim \mathcal{A}_{\text{SM}}(s, t, u) \cdot S(s, t, u) + T f(s, t, u) \cdot g(s, t, u),$$

where \mathcal{A}_{SM} is the SM amplitudes, $S(s, t, u)$ are the Veneziano amplitudes given in Eq. (9), T is the undetermined Chan-Paton parameter, $f(s, t, u)$ a kinematic function given in Eq. (10), and $g(s, t, u)$ some process-dependent kinematic functions. In the low-energy limit $s \ll M_S^2$, $S(s, t) \rightarrow 1$, $f(s, t, u) \rightarrow 0$, the open-string amplitude reduces to the standard model amplitude.

In this work we have performed a global analysis of the model based on the data from Drell-Yan production at the Tevatron, HERA neutral- and charged-current deep-inelastic scattering, and fermion pair production at LEP 2. Drell-Yan production at the Tevatron, due to large invariant mass data, provides the strongest constraint. Combining all data the string scale M_S must be ≥ 0.7 –2 TeV for Chan-Paton factors $|T| = 0$ –4 at 95% C.L. We have also estimated the expected sensitivity at the LHC, which is about 19 TeV for $|T| = 4$.

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- [1] P. Horava and E. Witten, Nucl. Phys. **B460**, 506 (1996).
 - [2] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); Phys. Rev. D **59**, 086004 (1999).
 - [3] J. D. Lykken, Phys. Rev. D **54**, R3693 (1996).
 - [4] G. Shiu and S. H. Tye, Phys. Rev. D **58**, 106007 (1998).
 - [5] K. Cheung, hep-ph/0409028.
 - [6] S. Cullen, M. Perelstein, and M. E. Peskin, Phys. Rev. D **62**, 055012 (2000).
 - [7] F. Cornet, J. I. Illana, and M. Masip, Phys. Rev. Lett. **86**, 4235 (2001).
 - [8] J. Friess, T. Han, and D. Hooper, Phys. Lett. B **547**, 31 (2002).
 - [9] P. Burikham, T. Han, F. Hussain, and D. McKay, Phys. Rev. D **69**, 095001 (2004).
 - [10] P. Burikham, F. Figy, and T. Han, Phys. Rev. D **71**, 016005 (2005); **71**, 019905 (2005).
 - [11] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
 - [12] E. Eichten, K. Lane, and M. E. Peskin, Phys. Rev. Lett. **50**, 811 (1983); V. Barger, K. Cheung, K. Hagiwara, and D. Zeppenfeld, Phys. Rev. D **57**, 391 (1998).
 - [13] K. Cheung, Phys. Lett. B **517**, 167 (2001).
 - [14] J. E. Paton and H. M. Chan, Nucl. Phys. **B10**, 516 (1969).

- [15] G. Veneziano, *Nuovo Cimento A* **57**, 190 (1968); *Phys. Rep.* **9**, 199 (1974).
- [16] H. L. Lai *et al.* (CTEQ Collaboration), *Eur. Phys. J. C* **12**, 375 (2000).
- [17] T. Affolder *et al.* (CDF Collaboration), *Phys. Rev. Lett.* **87**, 131802 (2001).
- [18] B. Abbott *et al.* (D0 Collaboration), *Phys. Rev. Lett.* **82**, 4769 (1999).
- [19] S. Chekanov *et al.* (ZEUS Collaboration), *Eur. Phys. J. C* **32**, 1 (2003).
- [20] S. Chekanov *et al.* (ZEUS Collaboration), *Phys. Rev. D* **70**, 052001 (2004).
- [21] C. Adloff *et al.* (H1 Collaboration), *Eur. Phys. J. C* **30**, 1 (2003).
- [22] S. Chekanov *et al.* (ZEUS Collaboration), *Phys. Lett. B* **539**, 197 (2002); **552**, 308 (2003).
- [23] S. Chekanov *et al.* (ZEUS Collaboration), *Eur. Phys. J. C* **28**, 175 (2003).
- [24] C. Adloff *et al.* (H1 Collaboration), *Eur. Phys. J. C* **19**, 269 (2001).
- [25] The LEP Collaborations: ALEPH Collaboration, DELPHI Collaboration, L3 Collaboration, OPAL Collaboration, the LEP Electroweak Working Group, the SLD Electroweak, Heavy Flavour Groups, hep-ex/0412015.
- [26] DELPHI Collaboration, <http://delphiwww.cern.ch/pubxx/conferences/beijing/>
- [27] OPAL Collaboration, <http://opal.web.cern.ch/Opal/pubs/ichep2004/abstr.html>
- [28] K. Cheung and G. Landsberg, *Phys. Rev. D* **62**, 076003 (2000).